AP Test Question 2007 Part A - With Calculator

3) The functions f and g are differentiable for all real numbers, and g is strictly increasing.
The table above gives the values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

$h(1) = 3$
$h(2) = 4$
$h(3) = -7$

Since f and g are differentiable, they must be continuous, thus h is continuous. Thus, by the Intermediate Value Theorem, h must take on all values of [3,-7] for r ε [1,3].

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b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

$h(1) = 3$
$h(3) = -7$

$$m = \frac{-7 - 3}{3 - 1} = -5$$

The Mean Value Theorem guarantees a c ε (1,3) such that $h(c) = -5$ because the slope of the secant line through $h(1)$ and $h(3)$ is equal to -5.

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c) Let w be the function give by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

$w'(3) = -2$

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d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

$$y = \frac{1}{5}x + \frac{3}{5}$$